invoked $\cos\phi(t)$ would be replaced by $\cos\phi(t) = 1$ and $\sin\phi(t) \simeq \phi(t)$. On the other hand, in order to describe the statistics of

$$Y = \frac{1}{T}\int_0^T \cos\phi(t)\,dt$$

$$\simeq 1 - \frac{1}{2T}\int_0^T \phi^2(t)\,dt$$

it has been necessary assume that $\phi(t)$ is gaussian. In principle, however, this assumption is not at all restrictive since the variance of the actual ϕ process at work in the loop can be substituted into the probability distribution $f_Y(y)$. Thus the nonlinear effects of the loop are taken into account.

Figures 16 and 17 illustrate the performance of a binary phase-shift-keyed communication system when $\phi(t)$ varies over the symbol interval. The signal-to-noise ratio $R = ST_0/N_0$ has been set such that the error probability of the system would be 10^{-3} (Fig. 16) and 10^{-5} (Fig. 17) in a perfectly synchronized system. For values of $4 \leq \delta \leq 5$ the results check, for all practical purposes, with those given previously (Ref. 1) where it is assumed that $\cos\phi$ is essentially constant over the symbol interval. For $\delta < 4$ the results presented here begin to deviate appreciably from those where $\cos\phi$ is assumed constant; hence, the model introduced here will be useful in designing and testing of phase-coherent systems which operate with $\delta < 4$, the low-rate end of the region of δ.

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G. Communications Systems Development: Efficiency of Noisy Reference Detection,

R. C. Tausworthe

1. Introduction

Lindsey (Ref. 1) has published results which, for a given modulation index, relate the observed signal-to-noise ratios (SNR) to equivalent signal losses caused by the noisy demodulation process. Recent measurements of the performance of the 8⅓-bits/s *Mariner* Mars 1969 engineering telemetry instituted a reevaluation of these analyses. The following report is a tabulation of the method for determining the demodulation efficiency as a function of loop phase error. Performance is then related to these efficiencies through the modulation index. This method was chosen because of the flexibility it affords when many indices, bandwidths, etc., are being considered.

2. Efficiency Equations

The output of a coherent amplitude detector is a process of the form (Fig. 18)

$$z(t) = P^{1/2} m(t)\, g(\phi) + n(t) \qquad (1)$$

in which $n(t)$ is wide-band noise normalized to have the same two-sided spectral density $N_0 = N_+/2$, as the input noise; $P = A^2$ is the rms detected sideband power; $g(\phi)$ is the detector phase characteristic, normalized so that $g(0) = 1$; and $m(t)$ is the detected modulation process, normalized so that $E(m^2(t)) = 1$. The modulation waveform we shall assume is one of M messages $\{m_k(t)\}$, for $0 \leq t < T$.

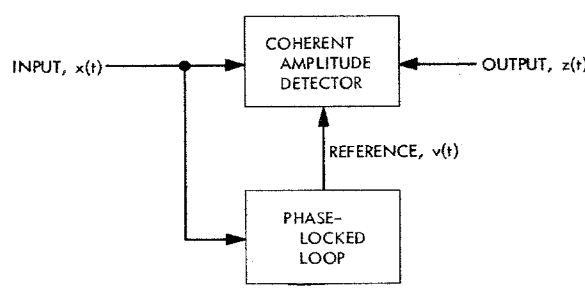

Fig. 18. Coherent detection by loop-derived reference

The process $z(t)$ is the input to a set of correlators, whose outputs at the end of a T-sec message are

$$w_k = \frac{A}{T}\int_0^T m(t)\, m_k(t)\, g(\phi(t))\,dt + N(T) \qquad (2)$$

$N(T)$ then is a gaussian random variable with variance

$$\sigma_N^2 = \frac{N_0}{T} \qquad (3)$$

We shall also assume that the phase-error process is derived from a phase-locked loop tracking the carrier or subcarrier. The loop bandwidth will be denoted w_L

and the phase-error density $p(\phi)$. The integral term of Eq. (2) can be written as a mean value plus a variational term

$$w_k = A(r_k \mu + \nu_k) + N(T) \tag{4}$$

in terms of the normalized cross-correlation r_k between the incoming message $m(t)$ and the kth stored one, $m_k(t)$, where $\mu = E[g(\phi)]$, the ν_k are random variables which depend on the value of k, and $\mathbf{v} = (\nu_1, \cdots, \nu_m)$ possesses some distribution $p(\mathbf{v})$. The actual overall probability of error is the average conditional error probability

$$\bar{P}_E = \int Pr[\text{error} | \nu_1, \cdots, \nu_m]\, p(\nu_1, \cdots, \nu_m)\, d\nu_1 \cdots d\nu_m \tag{5}$$

The difficulty in specifying the characteristics of $p(\nu_1, \cdots, \nu_m)$ lead to approximations for Eq. (5).

First, if the loop response is considerably more rapid than the integration time T (i.e., $\delta = 2/w_L T << 1$) then the correlator output tends to the average

$$\nu_k \approx 0 \tag{6}$$

in which case the outputs appear all to have an equivalent constant factor $E[g(\phi)]$ multiplying the signal amplitude A. Performance is then the same as it would be if the signal power were reduced by the factor $E^2[g(\phi)]$. The error rate will fit the usual maximum likelihood theory, giving rise to a probability of error as a function of the matched filter SNR parameter ρ_{mf}:

$$\rho_{mf} = \frac{PT}{N_+} E^2[g(\phi)] = RE^2[g(\phi)] \tag{7}$$

in which $R = PT/N_+$ is the undegraded value of ρ_{mf}. In this case, it is easy to see that the detector efficiency η is merely

$$\eta_0 = E^2[g(\phi)] = \mu^2 \tag{8}$$

A second approximation can be made when the loop is very sluggish with respect to the message (i.e., $\delta = 2/w_L T >> 1$). Then, over the interval $(0, T)$, the phase error is nearly constant (but randomly distributed according to $p(\phi)$). In this case, the correlator outputs are very nearly

$$w_k = Ar_k\, g(\phi) + N(T) \tag{9}$$

so that the averaged error probability yields the overall error rate

$$\bar{P}_E(R^{1/2}) = \int P_E(R^{1/2} g(\phi) | \phi)\, p(\phi)\, d\phi \tag{10}$$

The degradation is then clearly

$$\eta_\infty = \frac{1}{R}\{\bar{P}_E^{-1}[P_E(R^{1/2})]\}^2 \tag{11}$$

Because of the convexity of the function $Pr[E|\mathbf{v}]$, it follows that the actual efficiency is bounded by Eqs. (8) and (11):

$$\eta_\infty \leq \eta_\delta \leq \eta_0 \tag{12}$$

3. Error Probability

Since Eq. (11) requires it, let us consider the error probability function. For no coding and antipodal binary signals, the error rate is

$$P_E = \frac{1}{2}\operatorname{erfc}(R)^{1/2} \tag{13}$$

For orthogonal, equi-energy signals, the error rate is

$$\begin{aligned} P_E &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \exp(-y^2/2) \\ &\quad \times \left[\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{v+(2R)^{1/2}} \exp(-v^2/2)\, dv\right]^{M-1} dy \\ &\approx \frac{M-1}{2}\operatorname{erfc}\left(\frac{R}{2}\right)^{1/2} \end{aligned} \tag{14}$$

As a function of R, biorthogonal codes behave much the same (Ref. 2) as Eq. (14) indicates. Thus, for the three cases (no coding, orthogonal, biorthogonal) we have

$$P_E(R_1^{1/2}) \approx P_E(R_0^{1/2}) \frac{\operatorname{erfc}(\lambda R_1)^{1/2}}{\operatorname{erfc}(\lambda R_0)^{1/2}} \tag{15}$$

for any values R_0 and R_1 of R. The coefficient λ relates to the coding:

$$\lambda = \begin{cases} 1, & \text{uncoded} \\ \frac{1}{2}, & \text{coded (orthogonal/biorthogonal)} \end{cases} \tag{16}$$

We can set $R_0^{1/2} = R^{1/2}$ and $R_1^{1/2} = R^{1/2} g(\phi)$, above, to evaluate Eq. (10):

$$\overline{P}_E(R^{1/2}) = \frac{P_E(R^{1/2})}{\operatorname{erfc}(\lambda R)^{1/2}} \int_{-\pi}^{\pi} \operatorname{erfc}(\lambda R)^{1/2} g(\phi))\, p(\phi)\, d\phi \tag{17}$$

But the same error rate would occur if the value of R were R_{eq} and no degradation were allowed:

$$\overline{P}_E(R^{1/2}) = P_E(R_{eq}^{1/2}) = \frac{P_E(R^{1/2})}{\operatorname{erfc}(\lambda R)^{1/2}} \operatorname{erfc}(\lambda R_{eq})^{1/2} \tag{18}$$

It is then clear that

$$R_{eq} = \frac{1}{\lambda}\left\{\operatorname{erfc}^{-1}\left[\int_{-\pi}^{\pi} \operatorname{erfc}((\lambda R)^{1/2} g(\phi))\, p(\phi)\, d\phi\right]\right\}^2 \tag{19}$$

and, correspondingly, that

$$\eta_\infty = \frac{1}{\lambda R}\left\{\operatorname{erfc}^{-1}\left[\int_{-\pi}^{\pi} \operatorname{erfc}((\lambda R)^{1/2} g(\phi))\, p(\phi)\, d\phi\right]\right\}^2 \tag{20}$$

Thus, it remains only to evaluate Eqs. (8) and (20) for given $g(\phi)$ and $p(\phi)$ to obtain limits on η_δ.

4. Carrier-Extraction Degradation

For the carrier-extraction process the detector characteristic is

$$g(\phi) = \cos\phi \tag{21}$$

and the phase-error distribution, based on the first-order loop theory, is approximately (Ref. 3)

$$p(\phi) = \frac{\exp(\rho\cos\phi)}{2\pi I_0(\rho)} \tag{22}$$

in terms of the loop equivalent SNR (Ref. 4)

$$\rho = \frac{P_c}{N_0 w_L \Gamma} \tag{23}$$

Based on this ρ, a certain loop phase error σ^2 is present in the loop:

$$\sigma^2 \approx \frac{\pi}{3} + \frac{4}{I_0(\rho)} \sum_{n=1}^{\infty} \frac{(-1)^n I_n(\rho)}{n^2}$$
$$\sim \frac{1}{\rho}, \qquad \text{as } \rho \to \infty \tag{24}$$

It also follows that the degradation for $\delta << 1$ is

$$\eta_0 = \left(\frac{I_1(\rho)}{I_0(\rho)}\right)^2 \tag{25}$$

The value for η_∞ has been obtained by numerical integration, and appears along with the η_0 of Eq. (25) in Fig. 19, cross-plotted as a function of the loop error. It may be noted that when σ^2 is small, the two bounds converge approximately to the gaussian-phase-error result

$$1 - \eta_0 \sim \frac{1}{\rho} \sim \sigma^2 \sim 1 - \exp(-\sigma^2) \tag{26}$$

But as degradation becomes an appreciable percent, the two separate and depend not only on σ^2, but on λR as well. Because of the increasing steepness of P_E with λR, the degradation for $\delta >> 1$ becomes more drastic as λR increases. The degradation for $\delta << 1$ is, however, independent of λR.

5. Subcarrier-Loop Degradation

Assuming that the subcarrier is a square wave, the detector characteristic becomes triangular:

$$g(\phi) = 1 - \frac{2}{\pi}|\phi|, \qquad \text{for } |\phi| \leq \frac{3\pi}{2} \tag{27}$$

The approximate loop error density (based on a first-order loop) is again related (SPS 37-31, Vol. IV, pp. 311–325) to the loop equivalent SNR by

$$p(\phi) = \begin{cases} \dfrac{1}{C}\exp\left[-\dfrac{2\rho}{\pi^2}\phi^2\right], & \text{for } |\phi| \leq \pi/2 \\[2ex] \dfrac{1}{C}\exp\left[-\rho\left(1 - \dfrac{2(\phi-\pi)^2}{\pi^2}\right)\right], & \pi/2 \leq \dfrac{3\pi}{2} \end{cases}$$

$$C = \frac{\pi^{3/2}}{(2\rho)^{1/2}}\left\{\operatorname{erf}\left(\frac{\rho}{2}\right)^{1/2} + e^{-\rho} h\left(\frac{\rho}{2}\right)^{1/2}\right\} \tag{28}$$

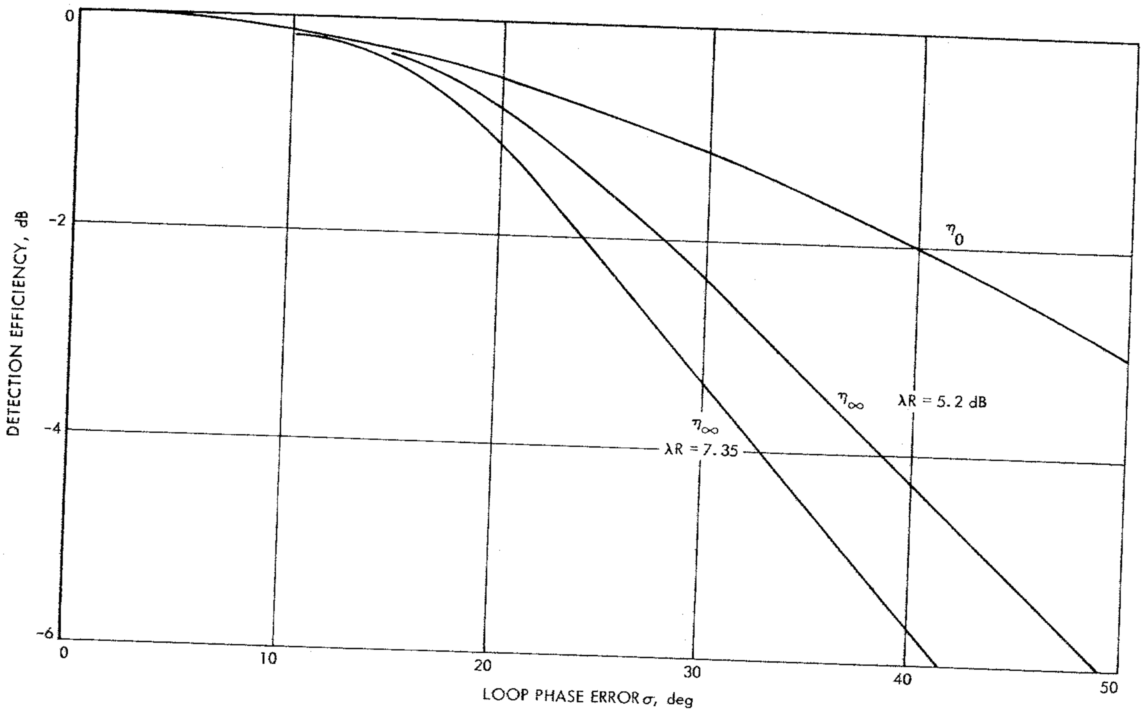

Fig. 19. Loop-derived reference degradation bounds for carrier tracking loop

where the imaginary error function

$$h(x) = \frac{2}{\pi^{1/2}} \int_0^x e^{t^2}\, dt$$

$$= \frac{2}{\pi^{1/2}} x \sum_{n=0}^{\infty} \frac{x^{2n}}{n!\,(2n+1)} \tag{29}$$

The phase-error variance is then

$$\sigma^2 = \left(\frac{\pi}{2}\right)^{5/2} \frac{\pi}{\rho^{3/2} C} \left\{ \operatorname{erf}\left[\left(\frac{\rho}{2}\right)^{1/2}\right] + (4\rho - 1)\, h\left[\left(\frac{\rho}{2}\right)^{1/2}\right] - 4\left(\frac{2\rho}{\pi}\right)^{1/2} e^{-\rho/2}\,(1 - e^{-\rho/2}) \right\} \tag{30}$$

and the low-rate efficiency can be straightforwardly obtained as

$$\eta_0 = \frac{\operatorname{erf}\left[\left(\frac{\rho}{2}\right)^{1/2}\right] - e^{-\rho} h\left[\left(\frac{\rho}{2}\right)^{1/2}\right]}{\operatorname{erf}\left[\left(\frac{\rho}{2}\right)^{1/2}\right] + e^{-\rho} h\left[\left(\frac{\rho}{2}\right)^{1/2}\right]} \times \left\{ 1 - \frac{(1 - e^{-\rho/2})^2}{\left(\frac{\pi\rho}{2}\right)^{1/2} \left\{ \operatorname{erfc}\left[\left(\frac{\rho}{2}\right)^{1/2}\right] - e^{-\rho} h\left[\left(\frac{\rho}{2}\right)^{1/2}\right] \right\}} \right\} \tag{31}$$

Again, the values for η_∞ have been obtained by numerical integration, and the two different behaviors plotted in Fig. 20 for comparison. As was evident in the previous case as well, the two degradations at low σ^2 behave like

$$\eta_0 = \left[1 - \left(\frac{2}{\pi}\right)^{3/2} \sigma\right]^2 \approx 1 - 2\left(\frac{2}{\pi}\right)^{3/2} \sigma \tag{32}$$

as would be predicted by a gaussian ϕ-process theory.

6. Interpolation Between η_0 and η_∞

For a given normalized code-word rate $\delta = 2/w_L T$, the actual efficiency η_δ lies between η_0 and η_∞. To compute η_δ exactly is an extremely difficult task, since the statistics of ϕ_k required by Eq. (4) are unknown. What we shall develop here is an interpolation formula for η_δ, rather than a direct evaluation of the efficiency. One very good approximation of the error probability in the vicinity of R_0 is obtained by a Taylor expansion of $\ln [P_E(R_1^{1/2})]$:

$$P_E(R_1^{1/2}) \approx P_E(R_0^{1/2})\, e^{\lambda(R_0 - R_1)} \tag{33}$$

for two comparative values R_0 and R_1 of $R = PT/N_+$, for both no-coding and orthogonal/biorthogonal coding, according to the value of λ. We take R_0 to be the value

$$R_0 = R\mu^2 = R\, E^2\,[g(\phi)] \tag{34}$$

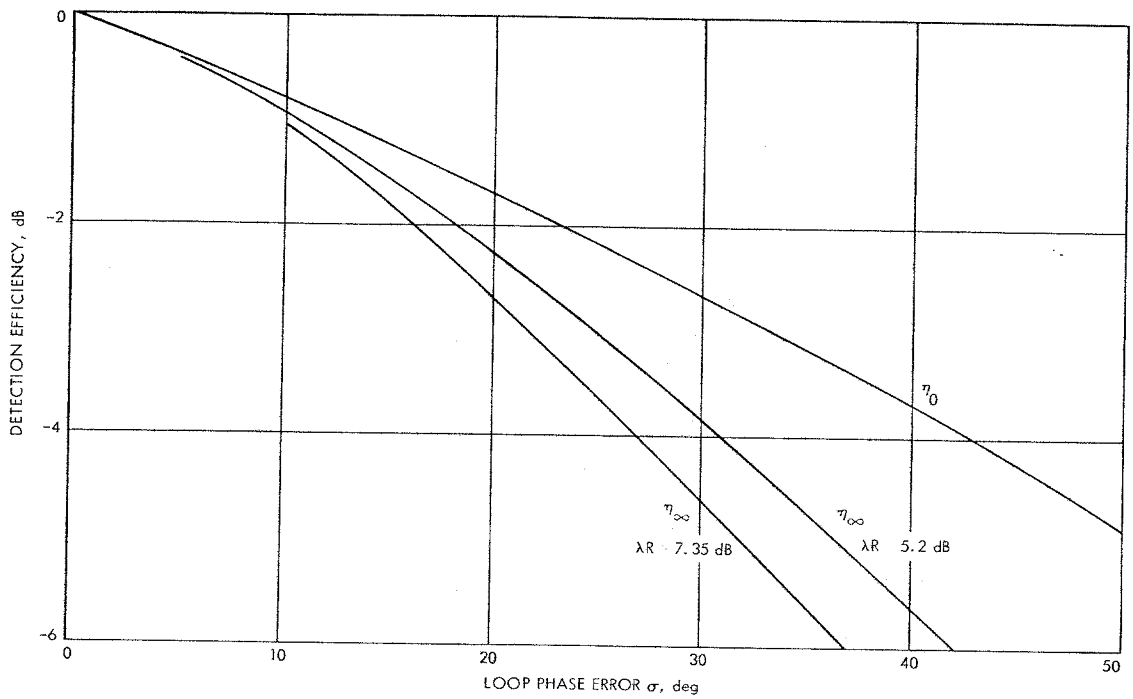

Fig. 20. Loop-derived reference degradation bounds for subcarrier-tone-tracking loop

corresponding to η_0, and take R_1 to be the apparent SNR corresponding to the correct word in Eq. (4):

$$R_1 = R(\mu + \nu)^2 \tag{35}$$

Then we can equate the observed error probability as having occurred with an equivalent value of $R = R_{eq}$, with no reference phase error:

$$\begin{aligned} P_E(R_{eq}^{1/2}) &= \int P_E(R_1^{1/2}|\nu)\,p(\nu)\,d\nu \\ &= P_E(R_0^{1/2}) \int \exp\left\{-\lambda R_0 \right. \\ &\quad \left. \times \left[\left(\frac{\nu}{\mu}\right)^2 + 2\left(\frac{\nu}{\mu}\right)\right]\right\} p(\nu)\,d\nu \end{aligned} \tag{36}$$

The form of P_E displayed in Eq. (33) then provides

$$\lambda(R_0 - R_1) = \ln \int \exp\left\{-\lambda R_0 \left[\left(\frac{\nu}{\mu}\right)^2 + 2\left(\frac{\nu}{\mu}\right)\right]\right\} p(\nu)\,d\nu$$

$$\begin{aligned} \lambda R(\eta_0 - \eta_\delta) &= \ln \int \left\{1 - \lambda R_0 \left[\left(\frac{\nu}{\mu}\right)^2 + 2\left(\frac{\nu}{\mu}\right)\right] \right. \\ &\quad \left. + \frac{\lambda^2 R_0}{2}\left[\left(\frac{\nu}{\mu}\right)^2 + 2\left(\frac{\nu}{\mu}\right)\right]^2 + \cdots \right\} p(\nu)\,d\nu \\ &= \ln\left\{1 - \lambda R_0 \left(\frac{\sigma_\nu}{\mu}\right)^2 + 2\lambda^2 R_0^2 \left(\frac{\sigma_\nu}{\mu}\right)^2 + \cdots\right\} \\ &= \left(\frac{\sigma_\nu}{\mu}\right)^2 [2\lambda^2 R_0^2 - \lambda R_0] + \cdots \end{aligned} \tag{37}$$

When σ_ν^2 is small, the first term will dominate the behavior of Eq. (37). Hence, as a result, we see that

$$\frac{\eta_0 - \eta_\delta}{\eta_0 - \eta_\infty} \approx \left(\frac{\sigma_\nu}{\sigma_{\nu,\infty}}\right)^2 = a \tag{38}$$

in which $\sigma_{\nu,\infty}^2$ is the variance of ν as $\delta \to \infty$. Hence, the interpolation formula we seek is

$$\eta_\delta = (1 - a)\,\eta_0 + a\eta_\infty \tag{39}$$

and is valid whenever Eq. (37) is dominated by its first term.

The parameter a defined by Eq. (38) involves only the expectation of the square of

$$\nu = \frac{1}{T}\int_0^T \{g(\phi(t)) - E[g(\phi)]\}\,dt \tag{40}$$

which is given straightforwardly by (Ref. 5)

$$\sigma_v^2 = \delta \int_0^{2/\delta} \left(1 - \frac{\delta x}{2}\right) [R_{g(\phi)}(x/w_L) - R_{g(\phi)}(\infty)]\, dx \tag{41}$$

The asymptotic values of σ_v^2 at very small and very large δ then verify our previous intuitive claim:

$$\sigma_v^2 \approx \begin{cases} \delta \displaystyle\int_0^{\infty} [R_{g(\phi)}(x/w_L) - R_{g(\phi)}(\infty)]\, dx, & \text{as } \delta \to 0 \\ \sigma_g^2, & \text{as } \delta \to \infty \end{cases} \tag{42}$$

It thus remains only to evaluate σ_v^2 at a particular value of δ. The ratio of the two variances σ_v^2 and σ_g^2 then give the parameter a. But because the loop is nonlinear, $R_{g(\phi)}(\tau)$ is not known, although there are several approximations available for calculation of $R_\phi(\tau)$. We can model $\phi(t)$ as a gaussian process having the same variance and bandwidth as the ϕ-process and thereby evaluate the autocorrelation of $g(\phi)$ in terms of that of $\phi(t)$ by Price's Theorem (Ref. 6). For example, if $g(\phi) = \cos\phi$, then

$$R_{\cos\phi}(\tau) - R_{\cos\phi}(\infty) = 2e^{-\sigma_\phi^2} \sinh^2\left(\frac{1}{2} R_\phi(\tau)\right)$$

$$\approx \frac{1}{2} e^{-\sigma_\phi^2} R_\phi^2(\tau) \tag{43}$$

while, if $g(\phi)$ is the triangular function present in square-wave subcarrier extraction, then

$$R_{\mathrm{tri}(\phi)}(\tau) - R_{\mathrm{tri}(\phi)}(\infty) =$$

$$\left(\frac{2}{\pi}\right)^3 \left[R_\phi(\tau) \sin^{-1}(R_\phi(\tau)) \sin^{-1}\left(\frac{R_\phi(\tau)}{\sigma_\phi^2}\right) + (\sigma_\phi^4 - R_\phi^2(\tau))^{1/2} - \sigma_\phi^2\right] \tag{44}$$

Further, we can model the correlation function of ϕ by the simple first-order loop result

$$R_\phi(\tau) = \sigma_\phi^2 \exp(-2w_L|\tau|) \tag{45}$$

and thereby evaluate the parameter a.

At best, the evaluation of a requires numerical integration. of σ_ϕ^2 is small in the carrier loop case, however, the approximation in Eq. (43) can be used to give

$$a\,(\text{carrier loop}) = \delta \int_0^{2/\delta} \left(1 - \frac{\delta x}{2}\right) e^{-4x}\, dx$$

$$= \frac{\delta}{4}\left[1 - \frac{\delta}{8}(1 - e^{-8/\delta})\right] \tag{46}$$

and is independent of σ_ϕ^2. This a is plotted in Fig. 21. The numerically integrated, more exact value at $\sigma_\phi^2 = 1$ is almost indistinguishable from Eq. (46).

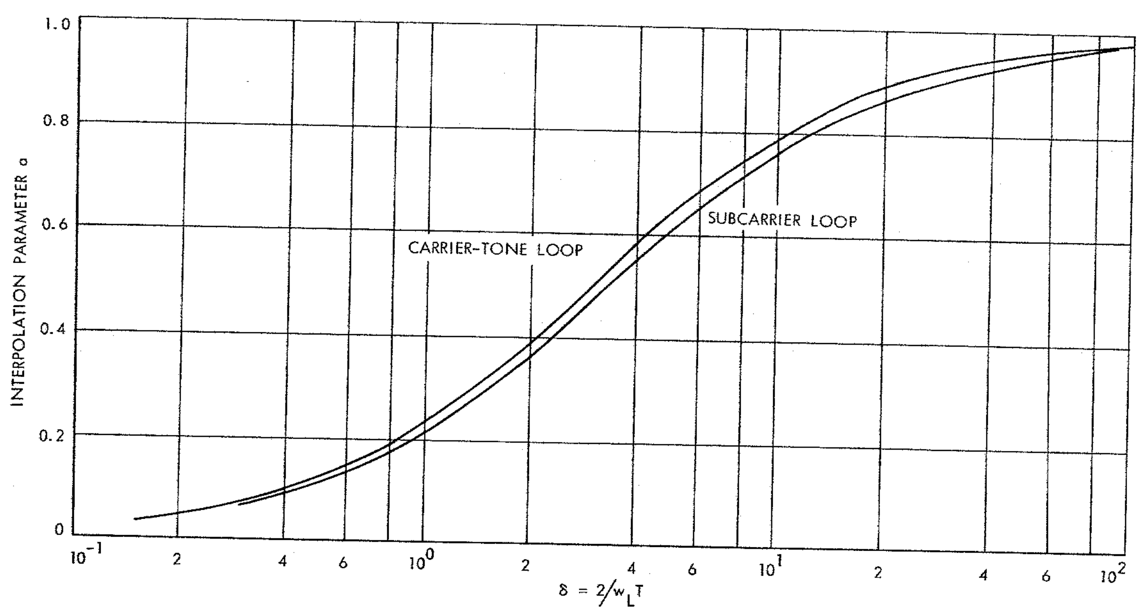

Fig. 21. Interpolation parameter a for sine wave (carrier tone) and square wave (subcarrier tone) loops as a function of the normalized data word rate

In the triangular subcarrier case, a reduces to the integral

$$a\,(\text{subcarrier}) = \frac{\delta}{\pi - 2}\int_0^{4/\delta}\left(1 - \frac{\delta x}{4}\right)[e^{-x}\sin^{-1}(e^{-x}) + (1 - e^{-2x})^{1/2} - 1]\,dx \quad (47)$$

which is also independent of σ_ϕ^2. The variation of a with δ is depicted also in Fig. 21. The approximate expression

$$a\,(\text{subcarrier loop}) = \frac{0.09135\,\delta + \delta^2}{1 + 3.3718\,\delta + \delta^2} \quad (48)$$

provides a simple formula for amazingly accurate results.

7. Conclusions

The efficiency of a coherent amplitude detector lies somewhere between limits set by two extreme theories, depending on the value of $\delta = 2/w_L T$. In the discussion we have considered (by assumption of the form $p(\phi)$ only) the effects of wide-band input noise. However, if there were other processes causing phase error, such as loop voltage-controlled oscillator noise, detection instabilities, etc., they can be considered as an equivalent phase-error term to be included in $p(\phi)$.

As long as the loop SNR, ρ, is greater than 10, $p(\phi)$ is very nearly gaussian, and a normal density can be substituted for $p(\phi)$, with the other instabilities reflected in the value of σ^2.

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H. Information Processing: Limiters in Phase-Locked Loops: A Correction to Previous Theory, *R. C. Tausworthe*

1. Introduction

In 1953 Davenport (Ref. 1) published a now-classic paper which showed that at very large values, the asymptotic output signal-to-noise ratio (SNR) of a limiter is twice its input SNR. Because of this, it was supposed that the same improvement ultimately should be evident in the performance of a phase-locked loop tracking the limiter output. In fact, the author (Ref. 2) used this result (erroneously, but subtly so) to derive a limiter performance factor Γ. Recently, however, G. D. Forney (Ref. 3) has presented a simple argument to show that the asymptotic factor of 2 is not realized in loop performance, although it is indeed present in output SNR. In this article, the author extends the asymptotic result to rederive the equivalent limiter performance factor.

2. Loop Theory and Noise Components

We shall assume (Fig. 22) that a loop has incident a sinusoid in wide-band noise, and we shall express this process in the form (Ref. 4)

$$y(t) = \alpha\, 2^{1/2}\sin(\omega_0 t + \theta) + n_Q(t)\, 2^{1/2}\sin(\omega_0 t + \hat{\theta}) + n_I(t)\, 2^{1/2}\cos(\omega_0 t + \hat{\theta}) \quad (1)$$

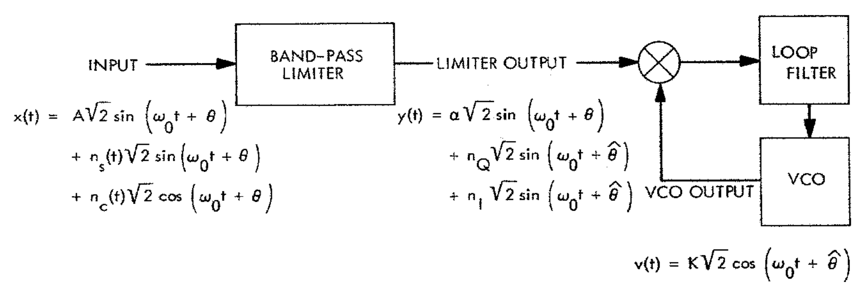

Fig. 22. The bandpass limiter phase-locked loop